

Money and risk aversion in a DSGE framework: a bayesian application to the Euro zone

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Abstract

In this paper, we set up and test a model of the Euro zone, with a special emphasis on the role of money. The model follows the New Keynesian DSGE framework, money being introduced in the utility function with a non-separability assumption. By using bayesian estimation techniques, we shed light on the determinants of output and inflation, but also of the interest rate, real money balances, flexible-price output and flexible-price real money balances variances. The role of money is investigated further. We find that its impact on output depends on the degree of agents' risk aversion, increases with this degree, and becomes significant when risk aversion is high enough. The direct impact of the money variable on inflation variability is essentially minor whatever the risk aversion level, the interest rate (monetary policy) being the overwhelming explanatory factor.

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1 Introduction

Standard New Keynesian literature analyses monetary policy practically without reference to monetary aggregates. In this now traditional framework, monetary aggregates do not explicitly appear as an explanatory factor neither in the output gap and inflation dynamics nor in interest rate determination. Inflation is explained by the expected inflation rate and the output gap. In turn, the output gap depends mainly on its expectations and the real rate of interest (Clarida, Galí and Gertler, 1999; Woodford, 2003; Galí and Gertler, 2007; Galí, 2008). Finally, the interest rate is established via a traditional Taylor rule in function of the inflation gap and the output gap.

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In this framework, monetary policy impacts aggregate demand and thus inflation and output, through change in the real interest rate. An increase in the interest rate reduces output, which increases the output gap, thus decreases inflation until a new equilibrium is reached. The money stock and money demand do not explicitly appear. The central bank sets the nominal interest rate so as to satisfy the demand for money (Woodford, 2003; Ireland, 2004).

This view of the transmission mechanism neglects the behavior of real money balances. First, there may exist a real balance effect on aggregate demand resulting from a change in prices. Second, as individuals re-allocate their portfolio of assets, the behavior of real money balances induces relative price adjustments on financial and real assets. In the process, aggregate demand changes, and thus output. By affecting aggregate demand, real money balances become part of the transmission mechanism. Hence, interest rate alone is not sufficient to explain the impact of monetary policy and the role played by financial markets (Meltzer, 1995, 1999, Brunner and Meltzer, 1968).

This monetarist transmission process may also imply a specific role to real money balances when dealing with risk aversion. When risk aversion increases, individuals may desire to hold more money balances to face the implied uncertainty and to optimize their consumption through time. Friedman alluded to this process as far back as 1956 (Friedman, 1956). If this hypothesis holds, risk aversion may influence the impact of real money balances on relative prices, financial assets and real assets, hence on aggregate demand and output.

Other considerations as to the role of money are worth mentioning. In a New Keynesian framework, the expected inflation rate or the output gap may "hide" the role of monetary aggregates, for example on inflation determination. Nelson (2008) shows that standard New Keynesian models are built on the strange assumption that central banks can control the long-term interest rate, while this variable is actually determined by a Fisher equation in which expected inflation depends on monetary developments. Reynard (2007) found that in the U.S. and the Euro area, monetary developments provide qualitative and quantitative information as to inflation. Assenlacher-Wesche and Gerlach (2006) confirm that money growth contains information about inflation pressures and may play an informational role as to the state of different non observed (or difficult to observe) variables influencing inflation or output.

How is money generally introduced in New Keynesian DSGE models? The standard way is to resort to money-in-the-utility (MIU) function, whereby real money balances are supposed to affect the marginal utility of consumption. Kremer, Lombardo, and Werner (2003) seem to support this non-separability assumption for Germany, and imply that real money balances contribute to the determination of output and inflation dynamics. A recent contribution introduces the role of money with adjustment costs for holding real balances, and shows that real money balances contribute to explain expected future variations of the natural interest rate in the U.S. and the Euro area (Andrés, López-Salido and Nelson, 2009). Nelson (2002) finds that money is a significant determinant of aggregate demand, both in the U.S. and in the U.K. However, the empirical work undertaken by Ireland (2004), Andrés, López-Salido, and Vallés (2006),

and Jones and Stracca (2008) suggests that there is little evidence as to the role of money in the cases of the United States, the Euro zone, and the UK.

Our paper differs in its empirical conclusion, giving a stronger role to money, at least in the Euro zone. It differs also somewhat in its theoretical set up. As in the standard way, we resort to money-in-the-utility function (MIU) with a non-separability assumption. Yet, in our framework, we specify *all* the micro-parameters. This specification permits to extract characteristics and implications of this type of model that cannot be extracted if only aggregated parameters are used. We will see, for example, that the coefficient of relative risk aversion plays a significant role in explaining the role of money.

Our model differs also in its inflation and output dynamics. Standard New Keynesian DSGE models give an important role to endogenous inertia on both output (consumption habits) and inflation (price indexation). In fact, both dynamics may have a stronger forward-looking component than an inertial component. And this appears to be the case at least in the Euro area, if not clearly in the U.S. (Galí, Gertler, and López-Salido, 2001). These inertial components may hide part of the role of money. Hence, our choice to remain as simple as possible on that respect in order to try to unveil a possible role for money balances.

We differ also from the empirical analyses of the Euro zone by using bayesian techniques in a New Keynesian DSGE framework like in Smets and Wouters (2007), while introducing money in the model. We also estimate all micro-parameters of the model, whereas current literature attempts to introduce money only by aggregating of some of these parameters, therefore leaving aside relevant information.

A simulation of the model is conducted in order to analyse the consequences of structural shocks. In the process we unveil transmission mechanisms generally neglected in traditional New Keynesian analyses. This framework highlights in particular the non-negligible role of money in explaining output variations, given a high enough risk aversion. It also highlights the overwhelming role of monetary policy in inflation variability.

The dynamic analysis of the model furthermore sheds light on the change in the role of money in explaining short run fluctuations in output as risk aversion changes. It shows that the higher the risk aversion, the higher the role of money in the transmission process.

Section 2 of the paper describes the theoretical set up. In Section 3, the model is calibrated and estimated with bayesian techniques and by using Euro area data. Impulse response functions and variance decompositions are analyzed in Section 4, with an emphasis on the impact of the coefficient of relative risk aversion. Section 5 concludes.

2 The model

The model consists of households that supply labor, purchase goods for consumption, hold money and bonds, and firms that hire labor and produce and

sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. This model is inspired by Galí (2008), Walsh (2003) and Smets and Wouters (2003).

2.1 Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t \left[\sum_{k=0}^{\infty} \beta^k U_{t+k} \right] \quad (1)$$

where U_t is the period utility function and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods represented by the interval $[0, 1]$. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures¹. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_t C_t + M_t + Q_t B_t \leq B_{t-1} + W_t N_t + M_{t-1} \quad (2)$$

for $t = 0, 1, 2, \dots$, where W_t is the nominal wage, P_t is an aggregate price index, N_t is hours of work (or the measure of household members employed), B_t is the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t + 1$ (each bond pays one unit of money at maturity and its price is Q_t where $i_t = -\log Q_t$ is the short term nominal rate) and M_t is the quantity of money holdings at time t . The above sequence of period budget constraints is supplemented with a solvency condition².

In the literature, utility functions are usually time-separable. To introduce an explicit role for money balances, we drop the assumption that household preferences are time-separable across consumption and real money balances. Preferences are measured with a CES utility function including real money balances. Under the assumption of a period utility given by

$$U_t = e^{\varepsilon_t^P} \left(\frac{1}{1-\sigma} \left((1-b) C_t^{1-\nu} + b e^{\varepsilon_t^M} \left(\frac{M_t}{P_t} \right)^{1-\nu} \right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi e^{\varepsilon_t^N} N_t^{1+\eta}}{1+\eta} \right) \quad (3)$$

consumption, labor, money and bond holdings are chosen to maximize (1) subject to (2) and the solvency condition. This CES utility function depends positively on the consumption of goods, C_t , positively on real money balances, M_t/P_t , and negatively on labour N_t . σ is the coefficient of relative risk aversion

¹See Appendix 6.1

²Such as $\forall t \lim_{n \rightarrow \infty} E_t[B_n] \geq 0$. It prevents engaging in Ponzi-type schemes.

of households (or the inverse of the intertemporal elasticity of substitution), ν is the inverse of the elasticity of money holdings with respect to the interest rate, and η is the inverse of the elasticity of work effort with respect to the real wage. The utility function also contains three structural shocks: ε_t^P is a general shock to preferences that affects the intertemporal substitution of households (preference shock), ε_t^m is a money demand shock and ε_t^n is a shock to the number of hours worked. All structural shocks are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term. b and χ are positive scale parameters.

This setting leads to the following conditions³, which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first order condition corresponding to the demand for contingent bonds implies that

$$\begin{aligned} \hat{c}_t = & E_t [\hat{c}_{t+1}] - \frac{1}{\nu - a_1 (\nu - \sigma)} (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) \\ & - \frac{(1 - a_1) (\nu - \sigma)}{\nu - a_1 (\nu - \sigma)} (E_t [\Delta \hat{m}_{t+1} - E_t [\hat{\pi}_{t+1}]]) + \xi_{t,c} \end{aligned} \quad (4)$$

where $\xi_{t,c} = -\frac{1}{\nu - a_1 (\nu - \sigma)} E_t [\Delta \varepsilon_{t+1}^P] - \frac{(1 - a_1) (\nu - \sigma)}{\nu - a_1 (\nu - \sigma)} \frac{1}{1 - \nu} E_t [\Delta \varepsilon_{t+1}^M]$ and by using the steady state of the first order conditions $a_1^{-1} = 1 + \left(\frac{b}{1-b}\right)^{\frac{1}{\nu}} (1 - \beta)^{\frac{\nu-1}{\nu}}$. The lowercase (\cdot) denotes the log-linearized (around the steady state) form of the original aggregated variables.

The demand for cash that follows from the household's optimization problem is given by

$$-\nu (\hat{m}_t - \hat{p}_t) + \nu \hat{c}_t + \varepsilon_t^M = a_2 \hat{i}_t \quad (5)$$

with $a_2 = \frac{1}{\exp(\frac{1}{\beta}) - 1}$ and where real cash holdings depend positively on consumption with an elasticity equal to unity and negatively on the nominal interest rate. In what follows we will take the nominal interest rate as the central bank's policy instrument. In the literature, due to the assumption that consumption and real money balances are additively separable in the utility function, cash holdings do not enter any of the other structural equations: accordingly, the above equation becomes recursive to the rest of the system of equations.

The first order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$\eta \hat{m}_t + (\nu - a_1 (\nu - \sigma)) \hat{c}_t - (\nu - \sigma) (1 - a_1) (\hat{m}_t - \hat{p}_t) + \xi_{t,n} = \hat{w}_t - \hat{p}_t \quad (6)$$

where $\xi_{t,n} = -\frac{(\nu - \sigma)(1 - a_1)}{1 - \nu} \varepsilon_t^M + \varepsilon_t^N$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (equation (4)), the intertemporal optimality condition setting the marginal rate of substitution between money and

³See Appendix 6.3

consumption equal to the opportunity cost of holding money (equation (5)), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage⁴ (equation (6)).

2.2 Firms

We assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good but uses an identical technology with the following production function⁵,

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (7)$$

where A_t is the level of technology, assumed to be common to all firms and to evolve exogenously over time, and α is the measure of decreasing returns.

All firms face an identical isoelastic demand schedule, and take the aggregate price level P_t and aggregate consumption index C_t as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t , only a fraction $1 - \theta$ of firms, with $0 < \theta < 1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation⁶.

2.3 Price dynamics

Let's assume a set of firms not reoptimizing their posted price in period t . Using the definition of the aggregate price level⁷ and the fact that all firms resetting prices choose an identical price P_t^* , leads to $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1 - \theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$. Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1 - \theta) (p_t^* - p_{t-1}) \quad (8)$$

In this setup, we don't assume inertial dynamics of prices. Inflation results from the fact that firms reoptimizing in any given period their price plans, choose a price that differs from the economy's average price in the previous period.

2.4 Price setting

A firm reoptimizing in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. This problem is solved and leads to a first-order Taylor expansion around the zero inflation steady state:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})] \quad (9)$$

⁴See Appendix 6.2

⁵For simplicity reasons, we assume a production function without capital.

⁶Thus, each period, $1 - \theta$ producers reset their prices, while a fraction θ keep their prices unchanged.

⁷As shown in Appendix 6.1

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \log(\varepsilon/(\varepsilon - 1))$ is the log of the desired gross markup.

2.5 Equilibrium

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0, 1]$ and all t . Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$; it follows that $Y_t = C_t$ must hold for all t . One can combine the above goods market clearing condition with the consumer's Euler equation (4) to yield the equilibrium condition

$$\begin{aligned} \hat{y}_t &= E_t[\hat{y}_{t+1}] - \frac{1}{\nu - a_1(\nu - \sigma)} (\hat{i}_t - E_t[\hat{\pi}_{t+1}]) \\ &\quad + \frac{(\sigma - \nu)(1 - a_1)}{\nu - a_1(\nu - \sigma)} (E_t[\Delta \hat{m}_{t+1}] - E_t[\hat{\pi}_{t+1}]) + \xi_{t,c} \end{aligned} \quad (10)$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i) di$. By using the production function (7) and taking logs, one can write the following approximate relation between aggregate output, employment and technology as

$$y_t = a_t + (1 - \alpha) n_t \quad (11)$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost:

$$mc_t = (\hat{w}_t - \hat{p}_t) - \widehat{mpn}_t \quad (12)$$

$$= (\hat{w}_t - \hat{p}_t) - \frac{1}{1 - \alpha} (\hat{a}_t - \alpha \hat{y}_t) \quad (13)$$

for all t , where \widehat{mpn}_t defines the economy's average marginal product of labor. As $mc_{t+k|t} = (\hat{w}_{t+k} - \hat{p}_{t+k}) - mpn_{t+k|t}$ we have

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1 - \alpha} (p_t^* - p_{t+k}) \quad (14)$$

where the second equality⁸ follows from the demand schedule combined with the market clearing condition $c_t = y_t$. Substituting (14) into (9) yields

$$p_t^* - p_{t-1} = (1 - \beta\theta) \Theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t[\widehat{mc}_{t+k}] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t[\pi_{t+k}] \quad (15)$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$.

⁸Note that under the assumption of constant returns to scale ($\alpha = 0$), $mc_{t+k|t} = mc_{t+k}$, i.e., the marginal cost is independent of the level of production and, hence, is common across firms.

Finally, (8) and (15) yield the inflation equation

$$\pi_t = \beta E_t [\pi_{t+1}] + \lambda_{mc} \widehat{mc}_t \quad (16)$$

where β , $\lambda_{mc} = \Theta \frac{(1-\theta)(1-\beta\theta)}{\theta}$. λ_{mc} is strictly decreasing in the index of price stickiness θ , in the measure of decreasing returns α , and in the demand elasticity ε .

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. From (6) and (11), the average real marginal cost can be expressed as

$$\begin{aligned} mc_t = & \left(\nu - (\nu - \sigma) a_1 + \frac{\eta + \alpha}{1 - \alpha} \right) \hat{y}_t - \hat{a}_t \left(\frac{1 + \eta}{1 - \alpha} \right) \\ & + (\sigma - \nu) (1 - a_1) (\hat{m}_t - \hat{p}_t) + \xi_{t,n} \end{aligned} \quad (17)$$

Under flexible prices the real marginal cost is constant and equal to $mc = -\mu$. Defining the natural level of output, denoted by y_t^f , as the equilibrium level of output under flexible prices leads to

$$\begin{aligned} mc = & \left(\nu - (\nu - \sigma) a_1 + \frac{\eta + \alpha}{1 - \alpha} \right) \hat{y}_t^f - \hat{a}_t \left(\frac{1 + \eta}{1 - \alpha} \right) \\ & + (\sigma - \nu) (1 - a_1) \widehat{mp}_t^f + \xi_{t,n} \end{aligned} \quad (18)$$

where $\widehat{mp}_t^f = \hat{m}_t^f - \hat{p}_t^f$, thus implying

$$\hat{y}_t^f = v_a^y \hat{a}_t + v_m^y \widehat{mp}_t^f + v_c^y + v_{sm}^y \varepsilon_t^M + v_{sn}^y \varepsilon_t^N \quad (19)$$

where

$$\begin{aligned} v_a^y &= \frac{1 + \eta}{(\nu - (\nu - \sigma) a_1) (1 - \alpha) + \eta + \alpha} \\ v_m^y &= \frac{(1 - \alpha) (\nu - \sigma) (1 - a_1)}{(\nu - (\nu - \sigma) a_1) (1 - \alpha) + \eta + \alpha} \\ v_c^y &= -\frac{\mu (1 - \alpha)}{(\nu - (\nu - \sigma) a_1) (1 - \alpha) + \eta + \alpha} \\ v_{sm}^y &= \frac{(\nu - \sigma) (1 - a_1) (1 - \alpha)}{(\nu - (\nu - \sigma) a_1) (1 - \alpha) + \eta + \alpha} \frac{1}{1 - \nu} \\ v_{sn}^y &= -\frac{1 - \alpha}{(\nu - (\nu - \sigma) a_1) (1 - \alpha) + \eta + \alpha} \end{aligned}$$

We deduce from (10) that $\hat{i}_t^f = (\nu - (\nu - \sigma) a_1) E_t [\Delta \hat{y}_{t+1}^f]$ and by using (5) we obtain the following equation of real money balances under flexible prices

$$\widehat{mp}_t^f = v_{y+1}^m E_t [\hat{y}_{t+1}^f] + v_y^m \hat{y}_t^f + \frac{1}{\nu} \varepsilon_t^M \quad (20)$$

where $v_{y+1}^m = -\frac{a_2(\nu-(\nu-\sigma)a_1)}{\nu}$ and $v_y^m = 1 + \frac{a_2(\nu-(\nu-\sigma)a_1)}{\nu}$

Subtracting (18) from (17) yields

$$\widehat{mc}_t = \psi_x (\hat{y}_t - \hat{y}_t^f) + \psi_m (\widehat{mp}_t - \widehat{mp}_t^f) \quad (21)$$

where $\widehat{mp}_t = \hat{m}_t - \hat{p}_t$ is the log linearized real money balances around its steady state, \widehat{mp}_t^f is its flexible-price counterpart, $\psi_x = \nu - (\nu - \sigma) a_1 + \frac{\eta + \alpha}{1 - \alpha}$ and $\psi_m = (\sigma - \nu) (1 - a_1)$.

By combining (21) with (16) we obtain

$$\hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \kappa_x (\hat{y}_t - \hat{y}_t^f) + \kappa_m (\widehat{mp}_t - \widehat{mp}_t^f) \quad (22)$$

where $\hat{y}_t - \hat{y}_t^f$ is the *output gap*, $\widehat{mp}_t - \widehat{mp}_t^f$ is the *real money balances gap*,

$$\kappa_x = \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \left(\frac{1 - \theta}{\theta} \right) (1 - \beta \theta) \left(\nu - (\nu - \sigma) a_1 + \frac{\eta + \alpha}{1 - \alpha} \right)$$

and

$$\kappa_m = \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \left(\frac{1 - \theta}{\theta} \right) (1 - \beta \theta) (\sigma - \nu) (1 - a_1)$$

Then (22) is our first equation relating inflation to its one period ahead forecast, the output gap and real money balances.

The second key equation describing the equilibrium of the model is obtained by rewriting (10) so as to determine output

$$\hat{y}_t = E_t [\hat{y}_{t+1}] - \kappa_r (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) + \kappa_{mp} E_t [\Delta \widehat{mp}_{t+1}] + z_t^x \quad (23)$$

where $\kappa_r = \frac{1}{\nu - (\nu - \sigma) a_1}$, $\kappa_{mp} = \frac{(\sigma - \nu)(1 - a_1)}{\nu - a_1(\nu - \sigma)}$ and $z_t^x = \xi_{t,c} = \kappa_{sp} E_t [\Delta \varepsilon_{t+1}^P] + \kappa_{sm} E_t [\Delta \varepsilon_{t+1}^M]$ where $\kappa_{sp} = -\frac{1}{\nu - a_1(\nu - \sigma)}$ and $\kappa_{sm} = -\frac{(1 - a_1)(\nu - \sigma)}{\nu - a_1(\nu - \sigma)} \frac{1}{1 - \nu}$. (23) is thus a dynamic IS equation including the real money balances.

The third key equation describes the behavior of the money stock. From (5) we obtain

$$\widehat{mp}_t = \hat{y}_t - \kappa_i \hat{i}_t + z_t^m \quad (24)$$

where $\kappa_i = a_2/\nu$ and $z_t^m = \frac{1}{\nu} \varepsilon_t^M$.

The last equation determines the interest rate through a standard smoothed Taylor-type rule:

$$\hat{i}_t = (1 - \lambda_i) \left(\lambda_\pi (\hat{\pi}_t - \pi^*) + \lambda_x (\hat{y}_t - \hat{y}_t^f) \right) + \lambda_i \hat{i}_{t-1} + z_t^i \quad (25)$$

where λ_π and λ_x are policy coefficients reflecting the weight on inflation and on the output gap; the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing. z_t^i is an exogenous *ad hoc* shock accounting for fluctuations of the nominal interest rate such that $z_t^i = \rho_i z_{t-1}^i + \varepsilon_{i,t}$ with $\varepsilon_{i,t} \sim N(0; \sigma_i)$. To simplify, we assume that the target inflation rate is equal to zero, i.e. $\pi^* = 0$.

3 Estimation

As Schorfheide (1999) or Smets and Wouters (2003), we apply Bayesian techniques to estimate our DSGE model. Contrary to Ireland (2004) or Andr  s *et al.* (2006), we did not choose to estimate our model by using the maximum of likelihood because such computation hardly converges toward a global maximum.

3.1 DSGE model

Our model consists of six equations and six dependent variables: inflation, nominal interest rate, output, flexible-price output, real money balances and its flexible-price counterpart. Flexible-price output and flexible-price real money balances are completely determined by shocks: flexible-price output is mainly driven by technology shocks (whereas fluctuations in the output gap can be attributed to supply and demand shocks) whereas the flexible-price real money balances is mainly driven by money shocks and flexible-price output.

$$\hat{y}_t^f = v_a^y \hat{a}_t + v_m^y \widehat{mp}_t^f - v_c^y + v_{sm}^y \varepsilon_t^M + v_{sn}^y \varepsilon_t^N \quad (26)$$

$$\widehat{mp}_t^f = v_{y+1}^m E_t [\hat{y}_{t+1}^f] + v_y^m \hat{y}_t^f + \frac{1}{\nu} \varepsilon_t^M \quad (27)$$

$$\hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \kappa_x (\hat{y}_t - \hat{y}_t^f) + \kappa_m (\widehat{mp}_t - \widehat{mp}_t^f) \quad (28)$$

$$\hat{y}_t = E_t [\hat{y}_{t+1}] - \kappa_r (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) + \kappa_{mp} E_t [\Delta \widehat{mp}_{t+1}] \quad (29)$$

$$+ \kappa_{sp} E_t [\Delta \varepsilon_{t+1}^P] + \kappa_{sm} E_t [\Delta \varepsilon_{t+1}^M] \quad (30)$$

$$\widehat{mp}_t = \hat{y}_t - \kappa_i \hat{i}_t + \frac{1}{\nu} \varepsilon_t^M \quad (31)$$

$$\hat{i}_t = (1 - \lambda_i) \left(\lambda_\pi (\hat{\pi}_t - \pi^*) + \lambda_x (\hat{y}_t - \hat{y}_t^f) \right) + \lambda_i \hat{i}_{t-1} + \rho_i z_{t-1}^i + \varepsilon_{i,t} \quad (32)$$

where

$$\begin{aligned} v_a^y &= \frac{1+\eta}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha} & \kappa_x &= \frac{(1-\alpha)(1-\beta\theta)(\nu-(\nu-\sigma)a_1+\frac{\eta+\alpha}{1-\alpha})}{1-\alpha+\alpha\varepsilon} \frac{1-\theta}{\theta} \\ v_m^y &= \frac{(1-\alpha)(\nu-\sigma)(1-a_1)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha} & \kappa_m &= \frac{(1-\alpha)(1-\beta\theta)(\sigma-\nu)(1-a_1)}{1-\alpha+\alpha\varepsilon} \frac{1-\theta}{\theta} \\ v_c^y &= \frac{\log\left(\frac{\varepsilon}{\varepsilon-1}\right)(1-\alpha)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha} & \kappa_r &= \frac{1}{\nu-a_1(\nu-\sigma)} \\ v_{sm}^y &= \frac{(\nu-\sigma)(1-a_1)(1-\alpha)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha} \frac{1}{1-\nu} & \kappa_{mp} &= \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)} \\ v_{sn}^y &= -\frac{1-\alpha}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha} & \kappa_{sp} &= -\frac{1}{\nu-a_1(\nu-\sigma)} \\ v_{y+1}^m &= -\frac{a_2(\nu-(\nu-\sigma)a_1)}{\nu} & \kappa_{sm} &= -\frac{(1-a_1)(\nu-\sigma)}{\nu-a_1(\nu-\sigma)} \frac{1}{1-\nu} \\ v_y^m &= 1 + \frac{a_2(\nu-(\nu-\sigma)a_1)}{\nu} & \kappa_i &= \frac{a_2}{\nu} \end{aligned}$$

with $a_1 = \frac{1}{1 + \left(\frac{b}{1-b}\right)^{\frac{1}{\nu}} (1-\beta)^{\frac{\nu-1}{\nu}}}$ and $a_2 = \frac{1}{\exp(\frac{1}{\beta}) - 1}$.

A static analysis of these coefficients is provided in Appendix 6.7.

3.2 Euro Area data



In this model of the Euro zone, $\hat{\pi}_t$ is the log-linearized inflation rate measured as the quarter to quarter change in the GDP Deflator, \hat{y}_t is the log-linearized output measured as the quarter to quarter change in the GDP, and i_t is the short-term (3-month) nominal interest rate. These Data are extracted from the Euro Area Wide Model database (AWM) of Fagan, Henry and Mestre (2001). \hat{mp}_t is the log-linearized quarter to quarter growth rate of real money balances. We use the *M3* monetary aggregate from the OECD database. \hat{y}_t^f , the trend of the log-linearized output, and \hat{mp}_t^f , the trend of the log-linearized growth rate of real money balances, are completely determined by structural shocks.

Structural shocks (ε_t^P , ε_t^N , and ε_t^M), the exogenous component of the interest

rate (ε_t^i) and of the productivity (ε_t^a) are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term such as $\varepsilon_t^k = \mu_k \varepsilon_{t-1}^k + \omega_{k,t}$ where $\varepsilon_{k,t} \sim N(0; \sigma_k)$ for $k = \{P, N, M, i, a\}$.

3.3 Calibration and results

In order to obtain bayesian estimates of all the structural parameters of the model, we need to calibrate the mean and the probability distribution of these parameters. Following Smets and Wouters (2005) and Galí (2007), we linearize the equations describing the model around the steady state and we choose prior distributions for the parameters which are added to the likelihood function⁹; the estimation of the implied posterior distribution of the parameters is done using the Metropolis algorithm (see Smets and Wouters, 2007, and Adolfson *et al.*, 2007).

Table 1: Calibration and estimation of structural parameters¹⁰

	Law	Prior mean	Posterior deviation	Posterior mean	Standard deviation	Confidence interval
β	beta	0.99	0.005	0.9933	0.0026	[0.9879; 0.9988]
σ	normal	2.0	0.05	1.9746	0.0503	[1.8966; 2.0593]
ν	normal	1.2	0.05	1.3271	0.0378	[1.2647; 1.3915]
η	normal	1.0	0.05	1.0079	0.0498	[0.9219; 1.0884]
θ	beta	0.66	0.05	0.7645	0.0281	[0.7181; 0.8138]
ε	normal	6.0	0.05	6.0029	0.0500	[5.9226; 6.0867]
α	beta	0.33	0.05	0.3921	0.0539	[0.3050; 0.4792]
b	beta	0.4	0.05	0.3981	0.0508	[0.3209; 0.4810]
λ_i	beta	0.6	0.05	0.5346	0.0356	[0.4748; 0.5923]
λ_π	normal	3.5	0.05	3.5154	0.0499	[3.4329; 3.5965]
λ_x	normal	1.5	0.05	1.5288	0.0493	[1.4458; 1.6086]
ρ_a	beta	0.8	0.075	0.8600	0.0360	[0.7829; 0.9440]
ρ_i	beta	0.8	0.075	0.9892	0.0039	[0.9824; 0.9962]
ρ_p	beta	0.8	0.075	0.8841	0.0167	[0.8580; 0.9102]
ρ_m	beta	0.8	0.075	0.9389	0.0177	[0.9117; 0.9680]
ρ_n	beta	0.8	0.075	0.8105	0.0776	[0.6819; 0.9455]
σ_a	invgamma	1	1	0.8976	0.0914	[0.6269; 1.1568]
σ_i	invgamma	1	1	0.6118	0.0660	[0.4953; 0.7232]
σ_p	invgamma	5	1	7.2887	0.8491	[5.9397; 8.7002]
σ_m	invgamma	1	1	1.4097	0.1016	[1.2402; 1.5757]
σ_n	invgamma	1	1	1.2876	0.2204	[0.3410; 2.4605]

Results are based on 10 chains, each with 100000 draws based on the Metropolis algorithm.

Following standard conventions, we choose beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters

⁹The solution takes the form of a state-space model that is used to compute the likelihood function.

¹⁰See Appendix 6.5 for the prior and posterior distributions.

that need to be constrained to be greater than zero, and normal distributions in other cases. As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions and we choose a beta distribution for shock persistence parameters (as well as for the backward component of the Taylor rule) with 0.8 mean and 0.05 standard error. We estimate the model with 106 observations from 1980 (Q4) to 2007 (Q2) in order to avoid high volatility periods before 1980 and during the latest financial crisis.

The estimates of the macro-parameters (aggregated structural parameters) are

$$\begin{array}{ll}
v_a^y = 0.79327 & \kappa_x = 0.06334 \\
v_m^y = -0.02733 & \kappa_m = 0.00173 \\
v_c^y = 0.04376 & \kappa_r = 0.53741 \\
v_{sm}^y = 0.08356 & \kappa_{mp} = 0.06116 \\
v_{sn}^y = -0.23668 & \kappa_{sp} = -0.53741 \\
v_{y+1}^m = -0.80737 & \kappa_{sm} = -0.18698 \\
v_y^m = 1.8074 & \kappa_i = 0.43389
\end{array}$$

4 Interpretation

4.1 Impulse response functions

The impulse response functions of all structural shocks are as follows.

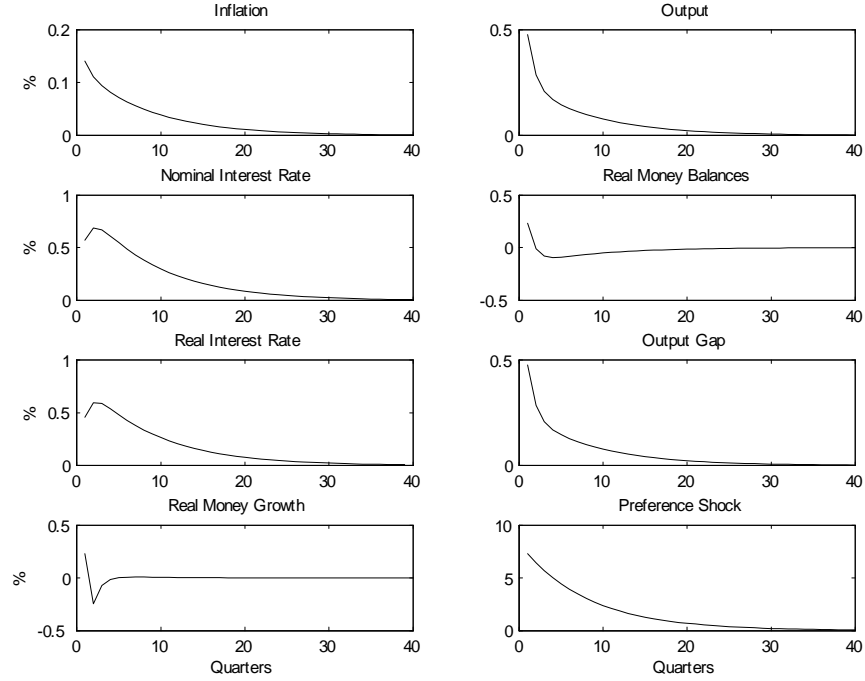


Figure 1: Preference shock

Figure 1 presents the response of key variables to a preference shock. In response to the shock, the inflation rate, the output, the output gap, real money balances, the nominal and the real rate of interest rise; real money growth displays a little overshooting process in the first periods, then returns quickly to its steady state value.

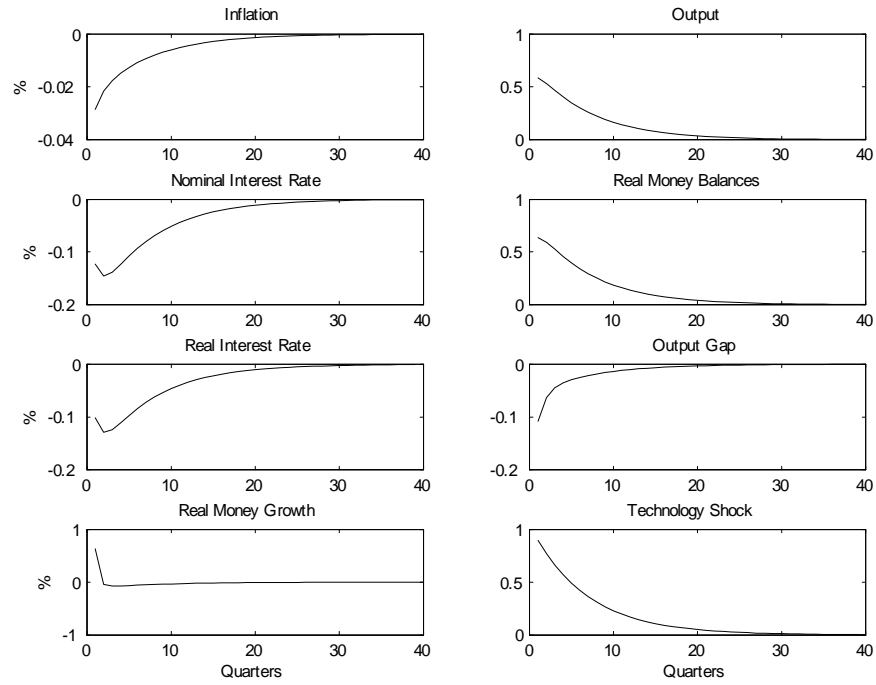


Figure 2: Technology shock

In Figure 2, we plot the response of the same variables to a technology shock. The output gap, the inflation, the nominal and the real interest rate decrease whereas output as well as real money balances and real money growth rise.

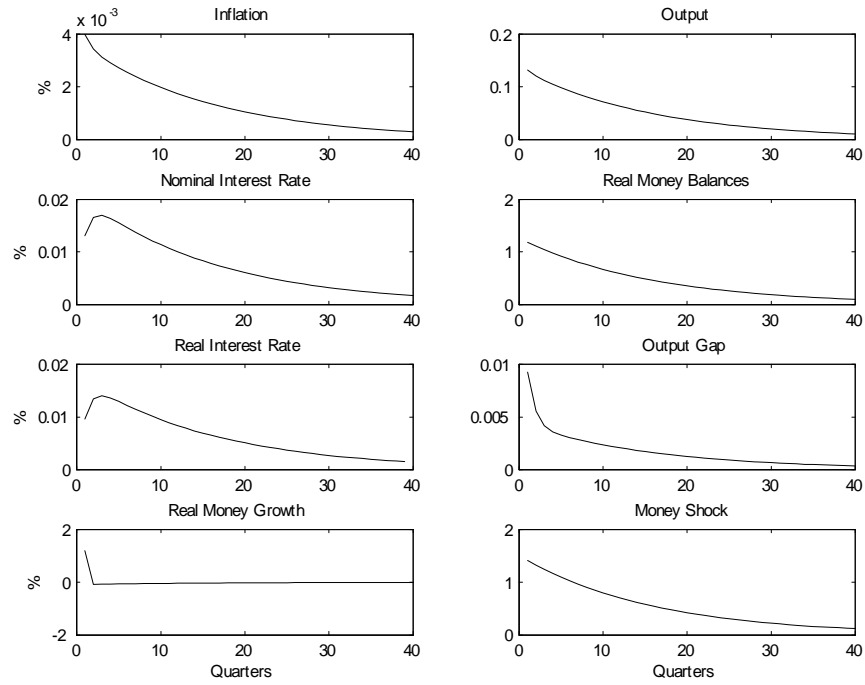


Figure 3: Money shock

Figure 3 exhibits the response to a money shock. Inflation, the nominal and the real rate of interest, the output and the output gap rise.

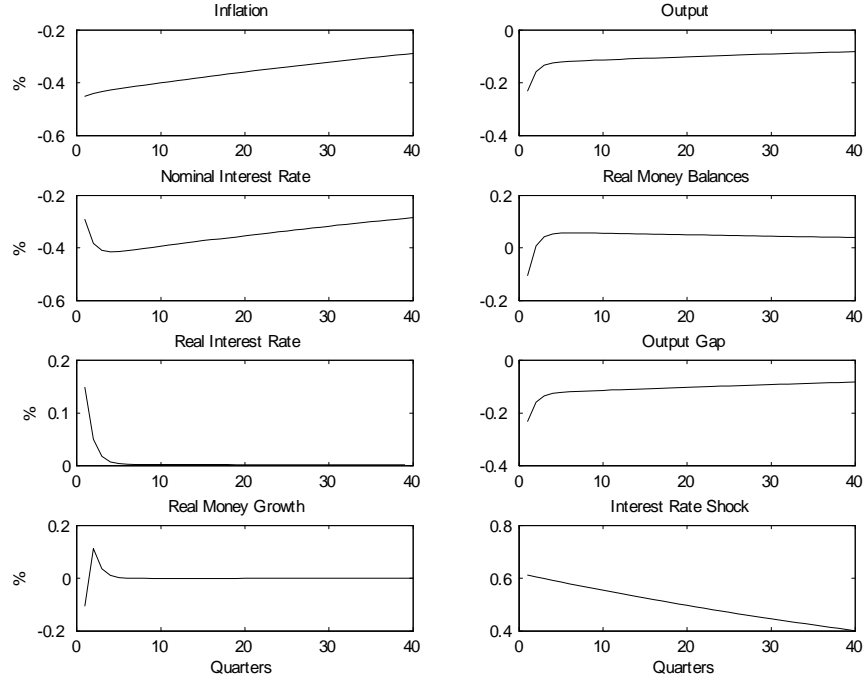


Figure 4: Interest rate shock

Figure 4 presents the response to an interest rate shock. Inflation, the nominal rate of interest, output and the output gap fall. The real rate of interest rises. A positive monetary policy shock induces a fall in interest rates due to a low enough degree of intertemporal substitution (i.e. the risk aversion parameter is high enough) which generates a large impact response of current consumption relative to future consumption. This result has been noted in, *inter alia*, Jeanne (1994) and Christiano et al. (1997).

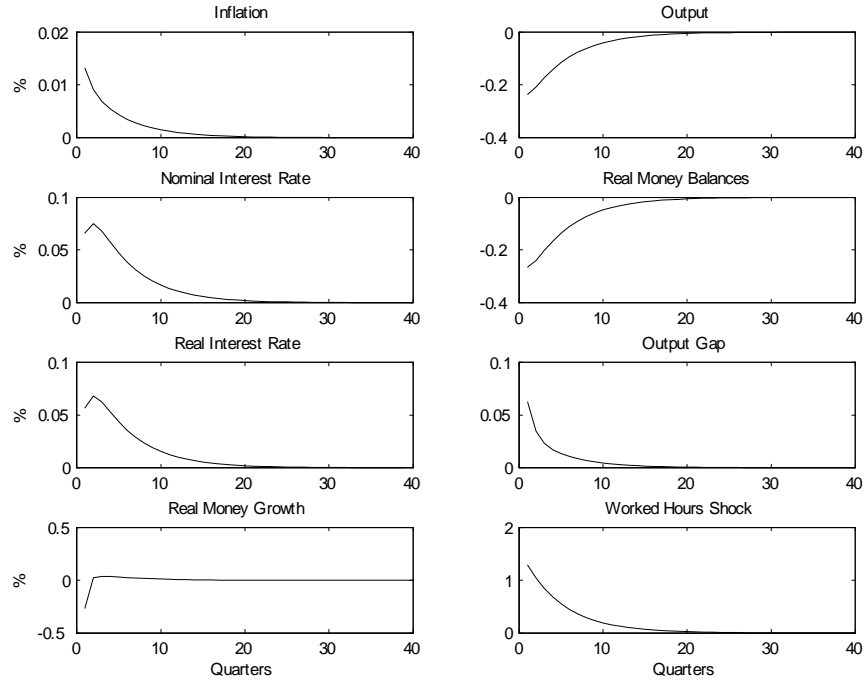


Figure 5: Labor shock

When there is a labor shock (Figure 5), inflation, the real and the nominal rate of interest, and the output gap increase. Output and real money balances decrease.

All these results are in line with the DSGE literature, especially with Galí (2007) and other studies on impulse response functions.

4.2 Variance decompositions

Here we analyze in two different ways the forecast error variance of each variable following exogenous shocks. The analysis is conducted first via an unconditional variance decomposition (Table 3), and second via a conditional variance decomposition (Figures 6 to 17).

Table 3: Unconditional variance decomposition (%)

	With $\sigma = 2$					With $\sigma = 6$				
	ε_t^N	ε_t^P	ε_t^i	ε_t^M	ε_t^a	ε_t^N	ε_t^P	ε_t^i	ε_t^M	ε_t^a
\hat{y}_t	6.19	15.6	25.7	4.47	48.1	0.20	10.1	22.6	17.0	50.2
$\hat{\pi}_t$	0.00	0.77	99.2	0.00	0.03	0.00	0.37	99.3	0.01	0.36
\hat{i}_t	0.21	25.5	73.3	0.03	0.98	0.14	21.1	67.7	0.26	10.8
\widehat{mp}_t	1.77	0.77	1.27	83.2	13.0	0.12	0.58	0.13	76.4	22.8
\hat{y}_t^f	11.8	0.00	0.00	5.67	82.6	0.55	0.00	0.00	16.5	82.9
\widehat{mp}_t^f	2.37	0.00	0.00	82.2	15.5	0.25	0.00	0.00	72.1	27.7

The unconditional variance decomposition shows that with a standard calibration of our model ($\sigma = 2$), about half of the variance of output results from the productivity shock, about a quarter from the interest rate shock, the remaining quarter from the other shocks. If money plays some role, this role is rather minor (an impact of less than 5%).

Yet, as Table 3 shows, the money shock contribution to the business cycle depends on the value of agents' risk aversion. Indeed, an estimation of our model with a higher risk aversion¹¹ ($\sigma = 6$) gives interesting information as to the role of money, and more generally as to the role of each shock.

Notably, it shows that a higher coefficient of relative risk aversion increases significantly the role of money in a business cycle.

¹¹See Appendix 6.4, Table 4.

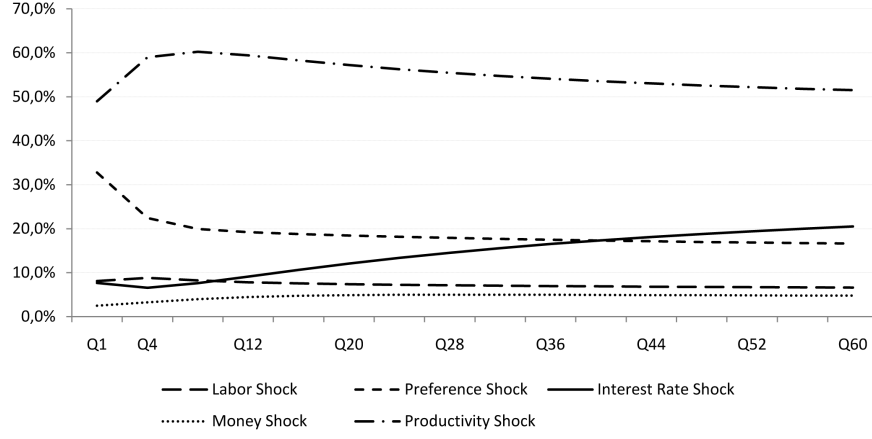


Figure 6: Forecast error variance decomposition of \hat{y}_t with $\sigma = 2$

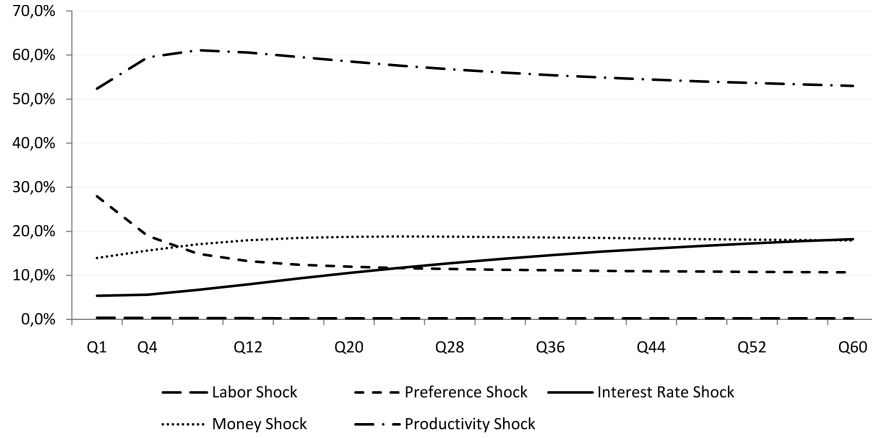


Figure 7: Forecast error variance decomposition of \hat{y}_t with $\sigma = 6$

If about half of the variance of output is still explained by the productivity shock, the role of the interest rate shock and especially the role of preference and labor shocks decrease notably whereas the impact of the money shock increases from about 4% to 17%, i.e. is multiplied by a factor of four.

The analysis through time (Figures 6 and 7) also shows that the impact of the money shock, and especially of the interest rate shock, increases a bit with the time horizon whereas it is the reverse for the preference shock.

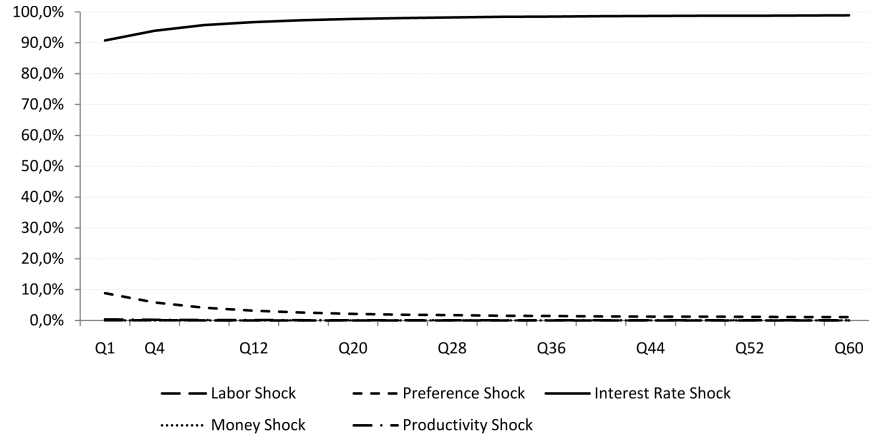


Figure 8: Forecast error variance decomposition of $\hat{\pi}_t$ with $\sigma = 2$

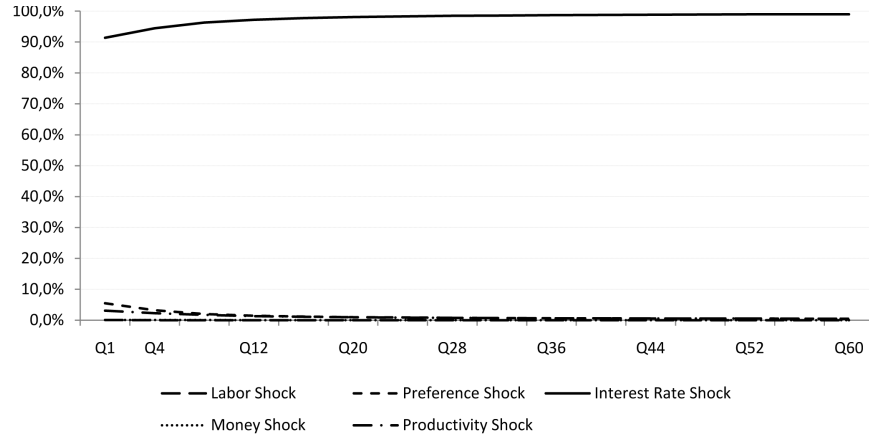


Figure 9: Forecast error variance decomposition of $\hat{\pi}_t$ with $\sigma = 6$

A look at the conditional and unconditional inflation variance decomposition shows the overwhelming role of the interest rate shock (the monetary policy shock) which explains more than 99% of the variance. It must be noted that the change in risk aversion (when σ goes from 2 to 6) does not affect this result, and there is no significant change of the respective impacts through time (Figures 8 and 9).

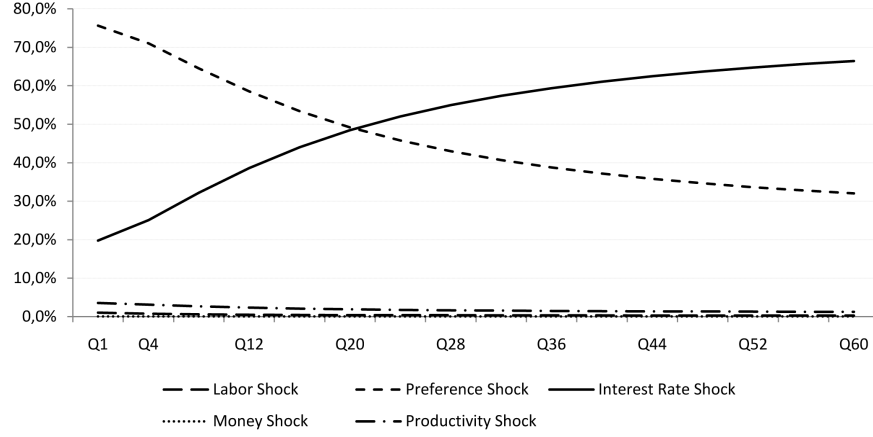


Figure 10: Forecast error variance decomposition of \hat{i}_t with $\sigma = 2$

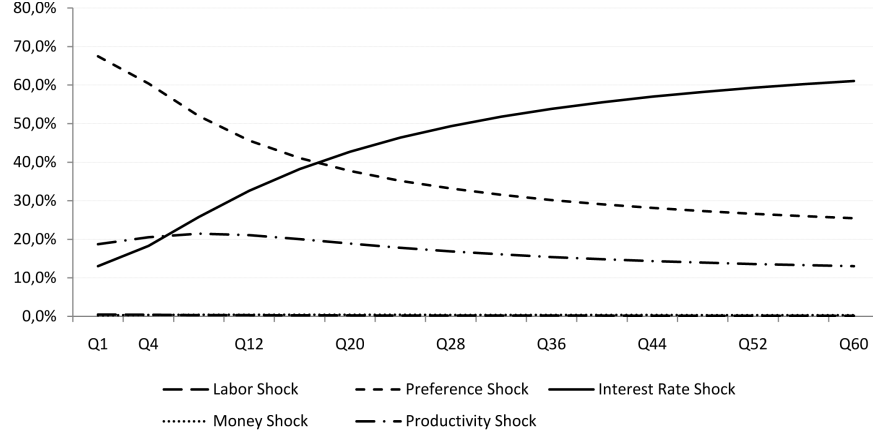


Figure 11: Forecast error variance decomposition of \hat{i}_t with $\sigma = 6$

The interest rate variance is dominated by the direct shock on the interest rate. Yet, as risk aversion increases, the role of the productivity shock increases. The relative importance of each of these shocks changes through time (Figures 10 and 11). Over short horizons, the preference shock explains almost 70% of the nominal interest rate variance whereas the interest rate shock explains less than 20%. For longer horizons, there is an inversion: the nominal interest rate shock explains close to 70% of the variance and the preference shock a bit more than 20%.

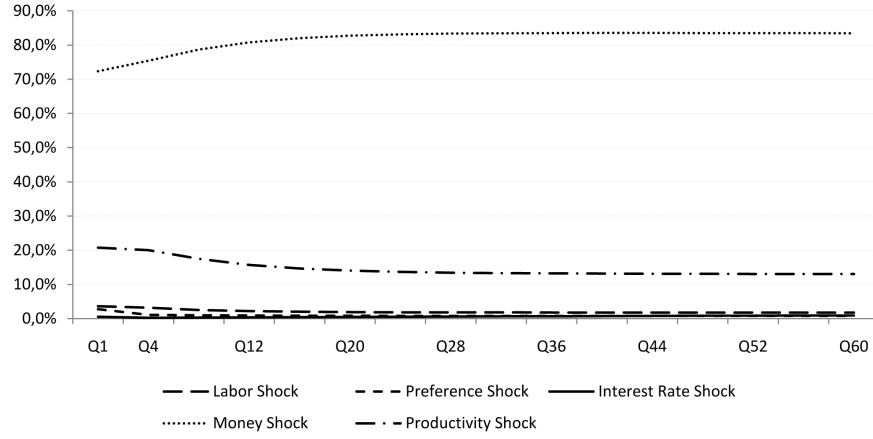


Figure 12: Forecast error variance decomposition of $\hat{m}p_t$ with $\sigma = 2$

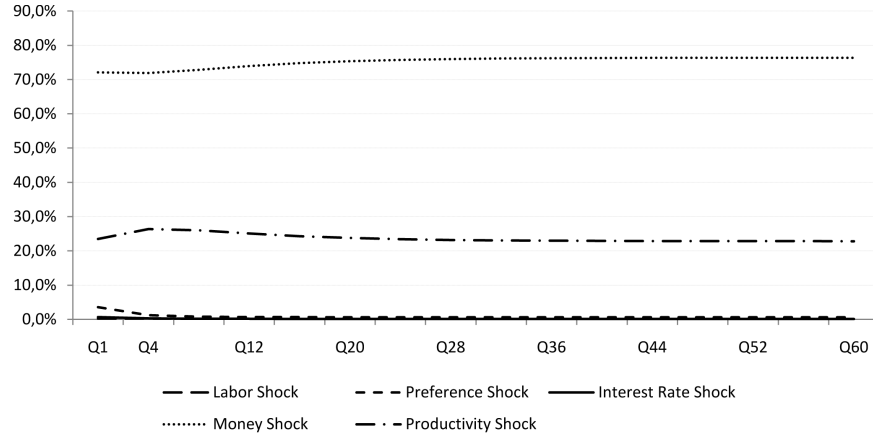


Figure 13: Forecast error variance decomposition of $\hat{m}p_t$ with $\sigma = 6$

Table 3 as well as Figures 12 and 13 show that real money balances are mainly explained by the real money balances shock and the productivity shock, with a small increase in the role of the productivity shock as risk aversion increases. The respective role of these two shocks barely changes through time.

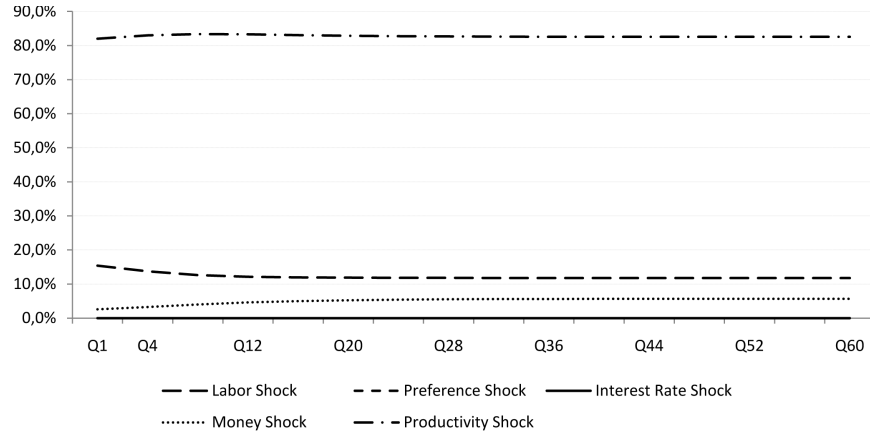


Figure 14: Forecast error variance decomposition of \hat{y}_t^f with $\sigma = 2$

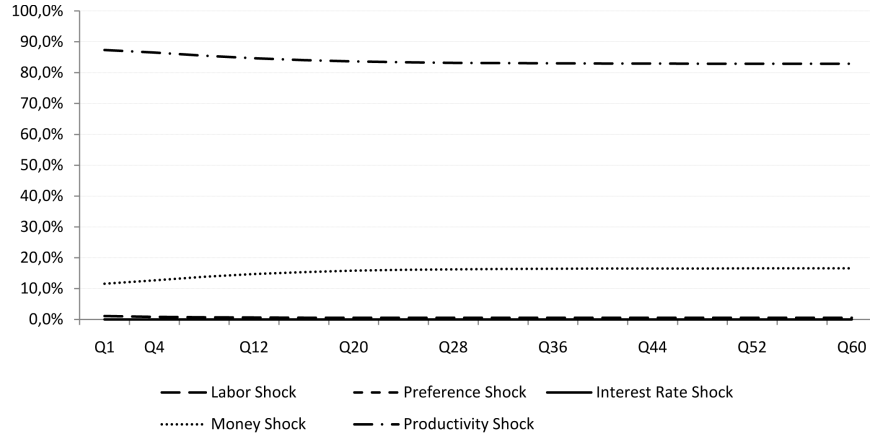


Figure 15: Forecast error variance decomposition of \hat{y}_t^f with $\sigma = 6$

It is also interesting to notice that the same type of analysis applies to the flexible-price output variance decomposition (Figures 14 and 15). Productivity is the main explanatory factor with a weight greater than 82%, the role of money increasing also with the relative risk aversion coefficient (from a weight of about 6% to almost 17%) whereas monetary policy plays no role and labor only a minor one.

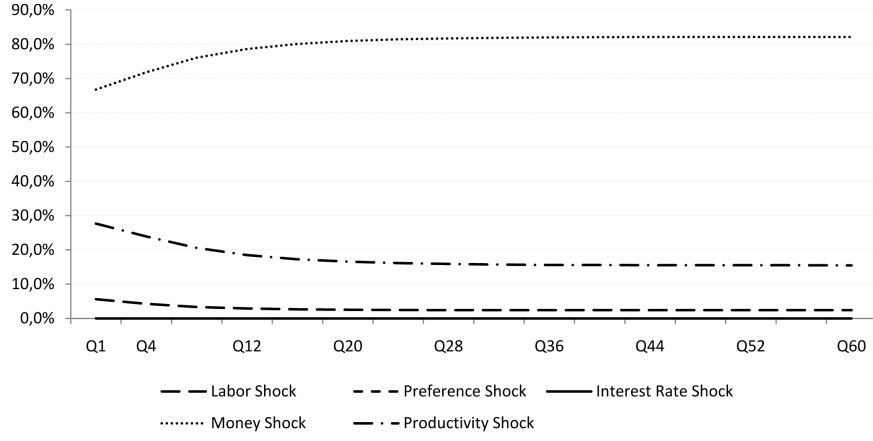


Figure 16: Forecast error variance decomposition of \widehat{mp}_t^f with $\sigma = 2$

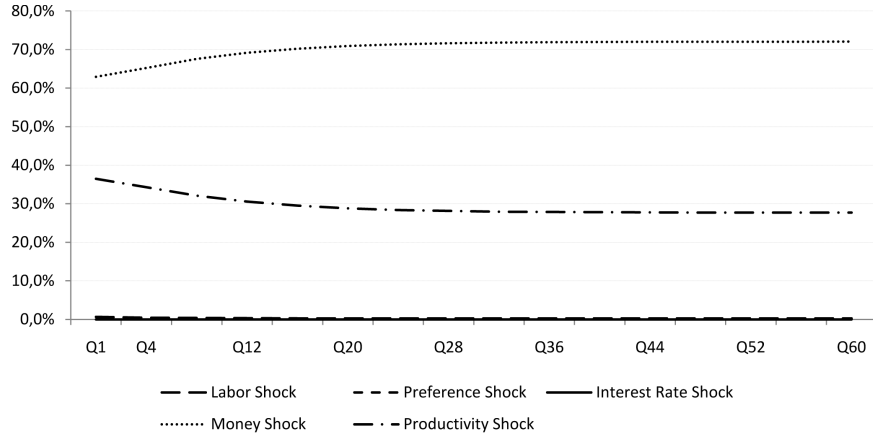


Figure 17: Forecast error variance decomposition of \widehat{mp}_t^f with $\sigma = 6$

As Figures 16 and 17 show, the flexible-price real balances variance is mainly explained by the money shock, with a significant impact of the productivity shock. The impact of each of these shocks does not vary much through time, but when risk aversion increases the impact of the productivity shock also increases.

5 Conclusion

In this paper, we built and empirically tested a model of the Euro zone, with a special emphasis on the role of money. The model follows the New Keynesian DSGE framework, but with money in the utility function whereby real money balances affect the marginal utility of consumption. By using bayesian estimation techniques, we shed light on the determinants of output and inflation, but also of interest rate, real money balances, flexible-price output and flexible-price real money balances variances. On that respect we further investigate the role of money, especially when intertemporal risk aversion changes.

Half of the variance of output is explained by the productivity shock, the other half by a combination of labor, preference, interest rate and money shocks. Almost the totality of the inflation variance is a consequence of the interest rate shock. The interest rate variance depends mainly on the interest rate shock, but the preference shock is also significant, as well as, to a lesser extent, the productivity shock. Real balances react essentially to money shocks, with a significant role left to the productivity shock. Interestingly, the flexible-price output variability depends strongly on the productivity shock, but the money shock remains significant. The flexible-price real balances variance is mainly explained by the money shock, with a significant impact of the productivity shock. These results are sensible and rather in line with prior expectations. We believe that this corroborates the credibility of our model.

To investigate further the role of each shock, especially the money shock, we calibrated the model with different risk aversion coefficients.

The first calibration of the model with a standard risk aversion shows that money plays a minor role in explaining output variability, a result in line with current literature (Andrès et *al.*, 2006; Ireland, 2004). Other calibrations with higher risk aversion imply that money plays a non-negligible role in explaining output fluctuations. And the more agents are risk averse, the higher the impact of money on output. This result differs from existing literature using New Keynesian DSGE frameworks with money, neglecting the role of a high enough risk factor.

On the other hand, the explicit money variable does not appear to have a notable direct role in explaining inflation variability, the overwhelming explanatory factor being the interest rate (monetary policy) whatever the level of risk aversion.

If these results are trustworthy, it can be inferred that the last financial crisis, by changing economic agents' perception of risks, may have increased the role of real money balances in the transmission mechanisms and in output changes.

6 Appendix

6.1 Aggregate consumption and price index

Let $C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ be a consumption index where $C_t(i)$ represents the quantity of good i consumed by the household in period t . This requires that C_t be maximized for any given level of expenditures $\int_0^1 P_t(i) C_t(i) di$ where $P_t(i)$ is the price of good i at time t . The maximization of C_t for any given expenditure level $\int_0^1 P_t(i) C_t(i) di = Z_t$ can be formalized by means of the Lagrangian

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda \left(\int_0^1 P_t(i) C_t(i) di - Z_t \right) \quad (33)$$

The associated first-order conditions are $C_t(i)^{-\frac{1}{\varepsilon}} C_t^{\frac{1}{\varepsilon}} = \lambda P_t(i)$ for all $i \in [0, 1]$. Thus, for any two goods (i, j) ,

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)} \right)^{-\varepsilon} \quad (34)$$

which can be substituted into the expression for consumption expenditures to yield $C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} \frac{Z_t}{P_t}$ for all $i \in [0, 1]$ where $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is an aggregate price index. The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t \quad (35)$$

Combining the two previous equations yields the demand schedule equation $C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$ for all $i \in [0, 1]$.

6.2 Optimization problem

Our Lagrangian is given by

$$L_t = E_t \left[\sum_{k=0}^{\infty} \beta^k U_{t+k} - \lambda_{t+k} V_{t+k} \right] \quad (36)$$

where

$$V_t = C_t + \frac{M_t}{P_t} + Q_t \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} - \frac{W_t}{P_t} N_t - \frac{M_{t-1}}{P_t} \quad (37)$$

and

$$U_t = e^{\varepsilon_t^P} \left(\frac{1}{1-\sigma} X_t^{1-\sigma} - \frac{\chi e^{\varepsilon_t^N} N_t^{1+\eta}}{1+\eta} \right) \quad (38)$$

where $X_t = \left((1-b) C_t^{1-\nu} + b e^{\varepsilon_t^M} \left(\frac{M_t}{P_t} \right)^{1-\nu} \right)^{\frac{1}{1-\nu}}$ is the non-separable part of the utility function.

The first order condition related to consumption expenditures is given by

$$\lambda_t = e^{\varepsilon_t^P} (1-b) C_t^{-\nu} X_t^{\nu-\sigma} \quad (39)$$

where λ_t is the Lagrange multiplier associated with the budget constraint at time t .

The first order condition corresponding to the demand for contingent bonds implies that

$$Q_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right] \quad (40)$$

The demand for cash that follows from the household's optimization problem is given by

$$b e^{\varepsilon_t^M} e^{\varepsilon_t^P} \left(\frac{M_t}{P_t} \right)^{-\nu} X_t^{\nu-\sigma} = \lambda_t - \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \quad (41)$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and inversely related to the nominal interest rate, as in conventional specifications.

$$\chi e^{\varepsilon_t^P} e^{\varepsilon_t^N} N_t^\eta = \lambda_t \frac{W_t}{P_t} \quad (42)$$

6.3 Log linearization

Log linearizing the Lagrangian multiplier (39) around its steady state yields

$$\hat{\lambda}_t = \varepsilon_t^P - \nu \hat{c}_t + (\nu - \sigma) \left(a_1 \hat{c}_t + (1 - a_1) \left(\hat{m}_t - \hat{p}_t + \frac{1}{1-\nu} \varepsilon_t^M \right) \right) \quad (43)$$

where $a_1 = \frac{(1-b)C^{1-\nu}}{(1-b)C^{1-\nu} + b \left(\frac{M}{P} \right)^{1-\nu}}$ is a constant term where C and $\frac{M}{P}$ are respectively consumption and real money balances at the steady state¹². We obtain from (39), (40) and (41) the following expression for a_1

$$a_1 = \frac{1}{1 + \left(\frac{b}{1-b} \right)^{\frac{1}{\nu}} (1-\beta)^{\frac{\nu-1}{\nu}}}$$

Log linearizing (40) around its steady state yields (with $Q_t = e^{-i_t}$)

$$-\hat{i}_t = E_t \left[\begin{aligned} & \Delta \varepsilon_{t+1}^P + (a_1 (\nu - \sigma) - \nu) \Delta \hat{c}_{t+1} \\ & + (1 - a_1) (\nu - \sigma) \left(\Delta \hat{m}_{t+1} - \Delta \hat{p}_{t+1} + \frac{1}{1-\nu} \Delta \varepsilon_{t+1}^M \right) - \hat{\pi}_{t+1} \end{aligned} \right] \quad (44)$$

¹²In order to determine (43), (44) and (46), we need to log linearize X_t around its steady state: $\hat{X}_t = a_1 \hat{c}_t + (1 - a_1) \left(\frac{1}{1-\nu} \hat{\varepsilon}_t^M + (\hat{m}_t - \hat{p}_t) \right)$

Log linearizing (41) around its steady state and up to an uninteresting constant yields

$$\varepsilon_t^M - \nu (\hat{m}_t - \hat{p}_t) + \nu \hat{c}_t = a_2 \hat{i}_t \quad (45)$$

where a_2 is such as $\widehat{1 - Q_t} = a_2 \hat{i}_t$ i.e. $a_2 = \frac{1}{\exp(\frac{1}{\beta}) - 1}$ because $\frac{1}{\beta}$ is the steady state interest rate.

Equation (45) is the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

Log linearizing (42) around its steady state yields

$$\eta \hat{m}_t - (a_1 (\nu - \sigma) - \nu) \hat{c}_t - (\nu - \sigma) (1 - a_1) \left(\hat{m}_t - \hat{p}_t + \frac{1}{1 - \nu} \varepsilon_t^M \right) + \varepsilon_t^N = \hat{w}_t - \hat{p}_t \quad (46)$$

Equation (46) is the condition for the optimal consumption-leisure arbitrage, implying that the marginal rate of substitution between consumption and labor is equated to the real wage.

6.4 Calibration and results ($\sigma = 6$)

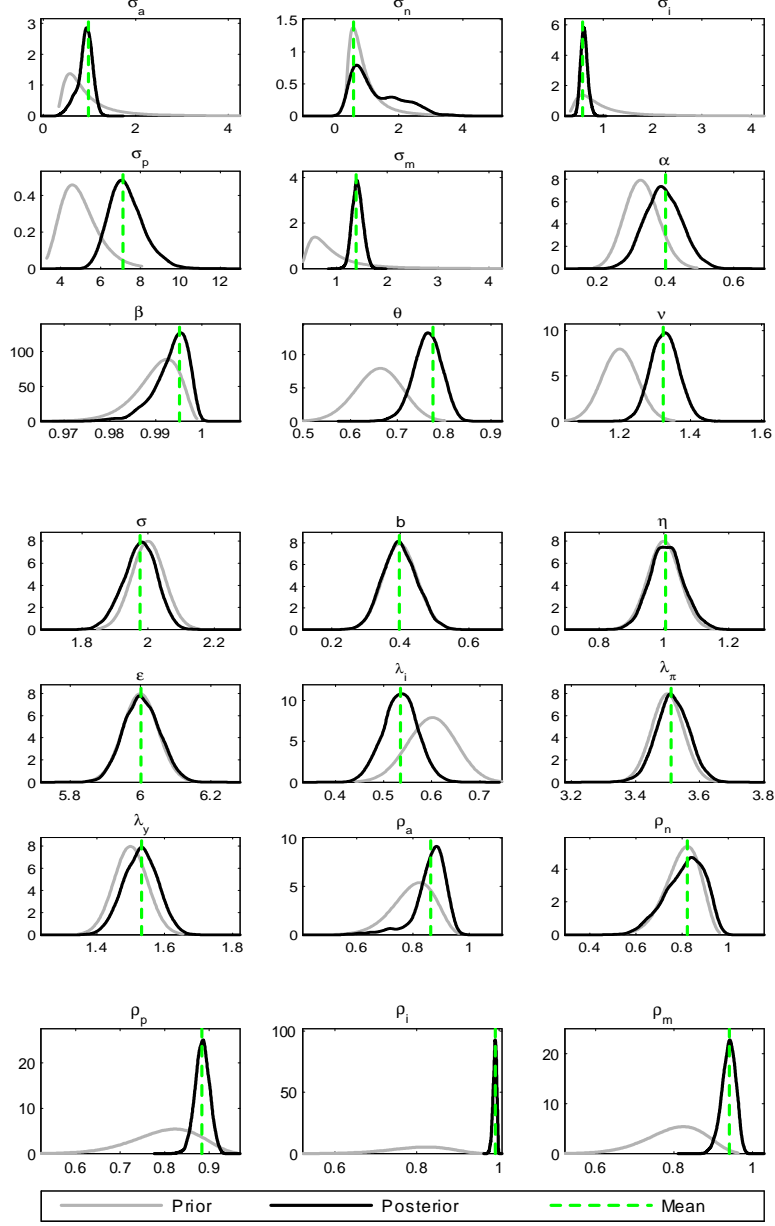
Table 4: Calibration and estimation of structural parameters¹³

	Law	Prior mean	Posterior deviation	Posterior mean	Standard deviation	Confidence interval
β	beta	0.99	0.005	0.9922	0.0030	[0.9860; 0.9985]
σ	normal	6.0	0.05	5.9906	0.0500	[5.9105; 6.0723]
ν	normal	1.2	0.05	1.4075	0.0357	[1.3493; 1.4678]
η	normal	1.0	0.05	1.0100	0.0498	[0.9322; 1.0996]
θ	beta	0.66	0.05	0.8224	0.0232	[0.7807; 0.8602]
ε	normal	6.0	0.05	6.0024	0.0500	[5.9211; 6.0829]
α	beta	0.33	0.05	0.4700	0.0583	[0.3794; 0.5631]
b	beta	0.4	0.05	0.3984	0.0508	[0.3180; 0.4752]
λ_i	beta	0.6	0.05	0.5781	0.0358	[0.5203; 0.6378]
λ_π	normal	3.5	0.05	3.5038	0.0499	[3.4128; 3.5846]
λ_x	normal	1.5	0.05	1.5280	0.0493	[1.4479; 1.6089]
ρ_a	beta	0.8	0.075	0.9006	0.0248	[0.8641; 0.9425]
ρ_i	beta	0.8	0.075	0.9882	0.0042	[0.9812; 0.9957]
ρ_p	beta	0.8	0.075	0.8251	0.0255	[0.7849; 0.8667]
ρ_m	beta	0.8	0.075	0.9352	0.0179	[0.9047; 0.9639]
ρ_n	beta	0.8	0.075	0.7987	0.0759	[0.6862; 0.9152]
σ_a	invgamma	1	1	2.1607	0.2511	[1.7459; 2.5606]
σ_i	invgamma	1	1	0.5743	0.0675	[0.4415; 0.6864]
σ_p	invgamma	5	1	7.7275	0.7471	[6.4409; 8.9630]
σ_m	invgamma	1	1	1.4870	0.1059	[1.3021; 1.6521]
σ_n	invgamma	1	1	0.9438	0.2150	[0.3243; 1.6599]

Results are based on 10 chains, each with 100000 draws based on the Metropolis algorithm.

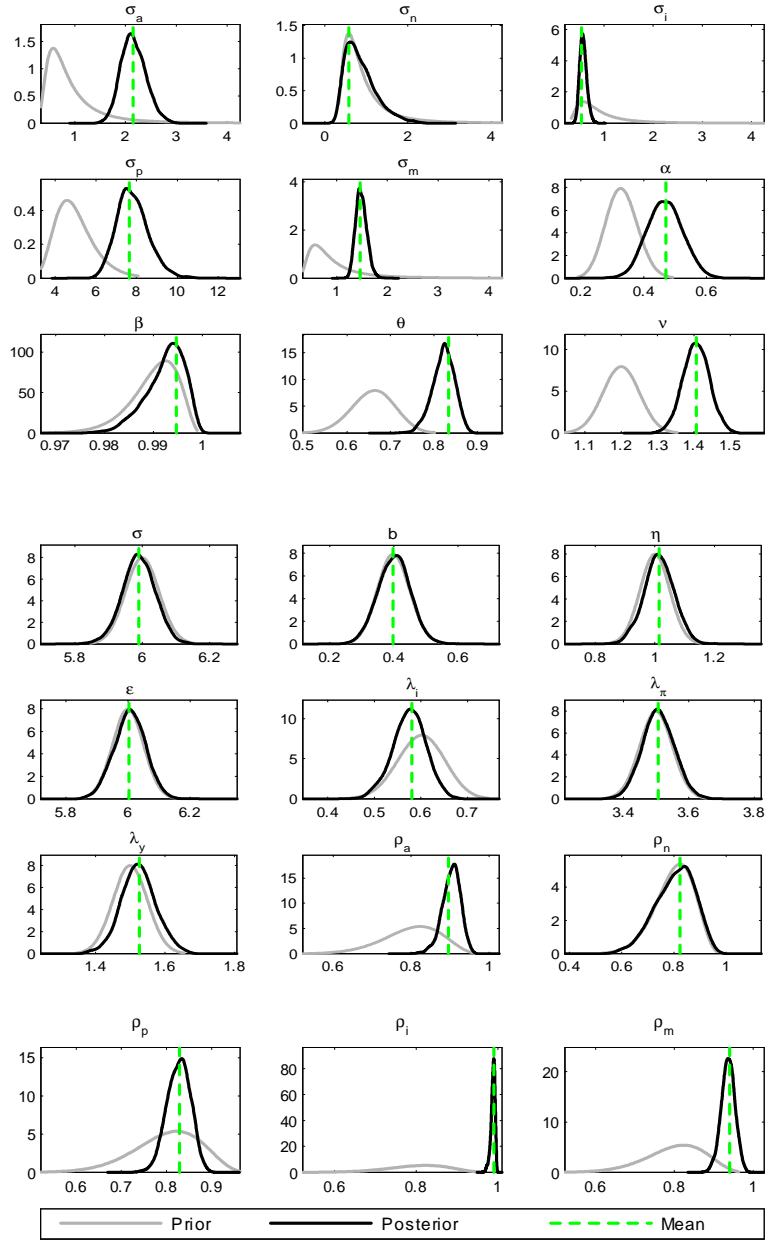
¹³See Appendix 6.6 for the prior and posterior distributions.

6.5 Priors and posteriors ($\sigma = 2$)



The vertical line denotes the posterior mode, the grey line is the prior distribution, and the black line is the posterior distribution.

6.6 Priors and posteriors ($\sigma = 6$)



6.7 Static analysis

The following table highlights the derivatives of the macro-coefficients with respect to σ , knowing that a_1 and a_2 are independent of σ , $\nu = 1.2$, and $\sigma \geq 2$.

	$\frac{\partial}{\partial \sigma}$
v_a^y	$\frac{-a_1(1-\alpha)(1+\eta)}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)^2} < 0$
v_m^y	$\frac{-(1-\alpha)(1-a_1)(\alpha+\eta+\nu(1-\alpha))}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)^2} < 0$
v_c^y	$\frac{-a_1 \log(\frac{\varepsilon}{\varepsilon-1})(1-\alpha)^2}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)^2} < 0$
v_{sm}^y	$-\frac{1}{1-\nu} \frac{(1-\alpha)(1-a_1)(\alpha+\eta+\nu(1-\alpha))}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)^2} > 0$
v_{sn}^y	$\frac{a_1(1-\alpha)^2}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)^2} > 0$
v_y^m	$\frac{a_1 a_2}{\nu} > 0$
v_{y+1}^m	$\frac{a_1 a_2}{\nu} < 0$
κ_x	$a_1 \frac{(1-\alpha)(1-\theta\beta)}{1-\alpha+\alpha\varepsilon} \frac{1-\theta}{\theta} > 0$
κ_m	$\frac{(1-\alpha)(1-a_1)(1-\theta\beta)}{1-\alpha+\alpha\varepsilon} \frac{1-\theta}{\theta} > 0$
κ_r	$-\frac{a_1}{(\nu-a_1(\nu-\sigma))^2} < 0$
κ_{mp}	$\frac{\nu(1-a_1)}{(\nu-a_1(\nu-\sigma))^2} > 0$
κ_{sp}	$\frac{a_1}{(\nu-a_1(\nu-\sigma))^2} > 0$
κ_{sm}	$\frac{1-a_1}{(\nu-a_1(\nu-\sigma))^2} \frac{\nu}{1-\nu} < 0$
κ_i	0

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